

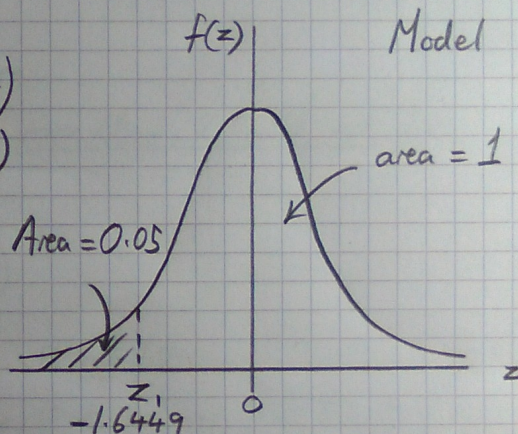
Statistics - AQA Question - A' Level

A machine produces metal rods of length L cm where L is normally distributed with standard deviation 0.2 cm

The rods are designed to fit into special grooves in a particular machine. Rods with a length between 20.4 cm and 20.9 cm will fit into the groove.

Given that 5% of the rods have a length less than 20.3 cm, find the probability that a randomly chosen rod will fit into the groove.

a) Model = $Z \sim N(\mu, \sigma^2)$
 $Z \sim N(0, 1^2)$



Real life = $X \sim N(\mu, 0.2^2)$

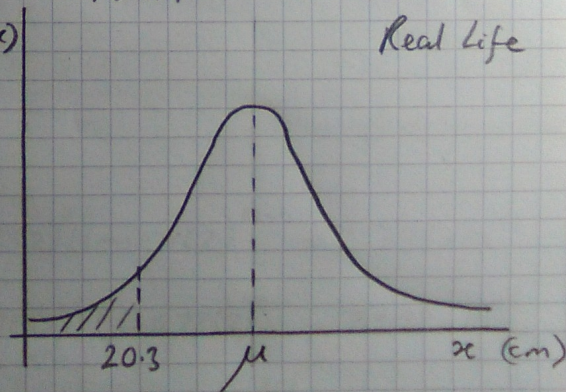
Conversion formula:

$$Z = \frac{X - \mu}{\sigma}$$

$$P(X < 20.3) = 0.05$$

$$\therefore P(Z < z_1) = 0.05$$

$$\therefore \underline{z_1 = -1.6449}$$



Using Inverse Normal cdf^{-1} on calculator

①

Using,

$$Z = \frac{X - \mu}{\sigma}$$

$$\mu = X - Z\sigma$$

$$\mu = 20.3 - (-1.6449 \times 0.2)$$

$$\mu = 20.629 \text{ cm}$$

$$P(20.4 < X < 20.9) = \underline{0.7862}$$

Using Normal CD
on calculator

b) 10 rods are chosen at random

Find the probability that more than 2 will not fit.

$$Y \sim B(n, p)$$

$$Y \sim B(10, 0.7862)$$

Probability that more than 2 will not fit

= Probability that no more than 2 will fit

$$P(X \leq 2) = \underline{1.2897 \times 10^{-4}}$$

Using Binomial CD
(Variable) on calculator.

Alternatively, using formula =

$$P(X = 0) = {}^{10}C_0 (0.7862)^0 (1 - 0.7862)^{10-0} = 1.996 \times 10^{-7}$$

$$P(X = 1) = {}^{10}C_1 (0.7862)^1 (1 - 0.7862)^{10-1} = 7.338 \times 10^{-6}$$

$$P(X = 2) = {}^{10}C_2 (0.7862)^2 (1 - 0.7862)^{10-2} = 1.214 \times 10^{-4}$$

$$\underline{1.2897 \times 10^{-4}}$$

②

c) A second machine produces rods of length M cm where M is normally distributed with standard deviation 0.15 cm. A random sample of 20 rods is taken from this machine and the sample mean is found to be 20.5 cm.

Stating your hypothesis clearly and using a 0.25% level of significance, test whether the mean length of these rods is greater than 20.4 cm.

$$H_0: \mu = 20.4 \text{ cm}$$

- Null hypothesis

$$H_1: \mu > 20.4 \text{ cm}$$

- Alternative hypothesis
(one tailed test)

$$\alpha = 0.0025$$

- significance level

$$X \sim N(20.4, 0.15^2)$$

- distribution

$$n = 20$$

- number in random sample

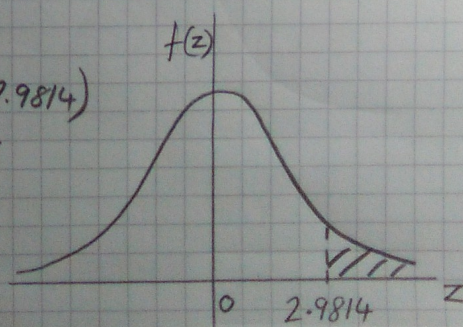
$$\bar{x} = 20.5 \text{ cm}$$

- test statistic

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{20.5 - 20.4}{0.15 / \sqrt{20}} = 2.9814 \dots$$

Method ① Finding the p value

$$\begin{aligned} P(Z > 2.9814) &= 1 - (P(Z < 2.9814)) \\ &= 1 - 0.9986 \\ &= 1.4 \times 10^{-3} \\ &= 0.0014 \end{aligned}$$



$$\text{As } 0.0014 < 0.0025$$

The result is significant & we reject the null hypothesis.

③

Method ② Finding the critical region

$$P(Z > z) = 0.0025$$

$$\therefore P(Z < -z) = 0.0025$$

$$-z < -2.8070$$

$$\underline{z > 2.8070}$$

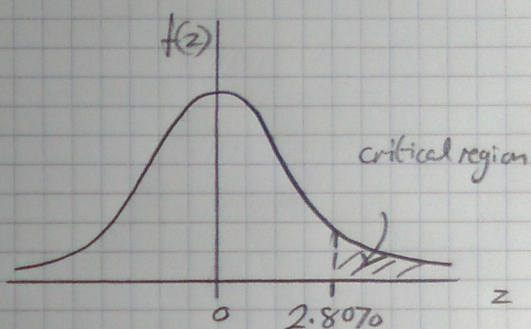
Using Inverse Normal
on calculator.

Critical region.

$$\text{Since } z = 2.9814 > 2.8070$$

The observed value of the test
statistic lies in the critical region.

So the result is significant.



Conclusion:

There is evidence at the 0.25% level of significance
to reject H_0 & to suggest that the mean length
has increased.

↑ include context.